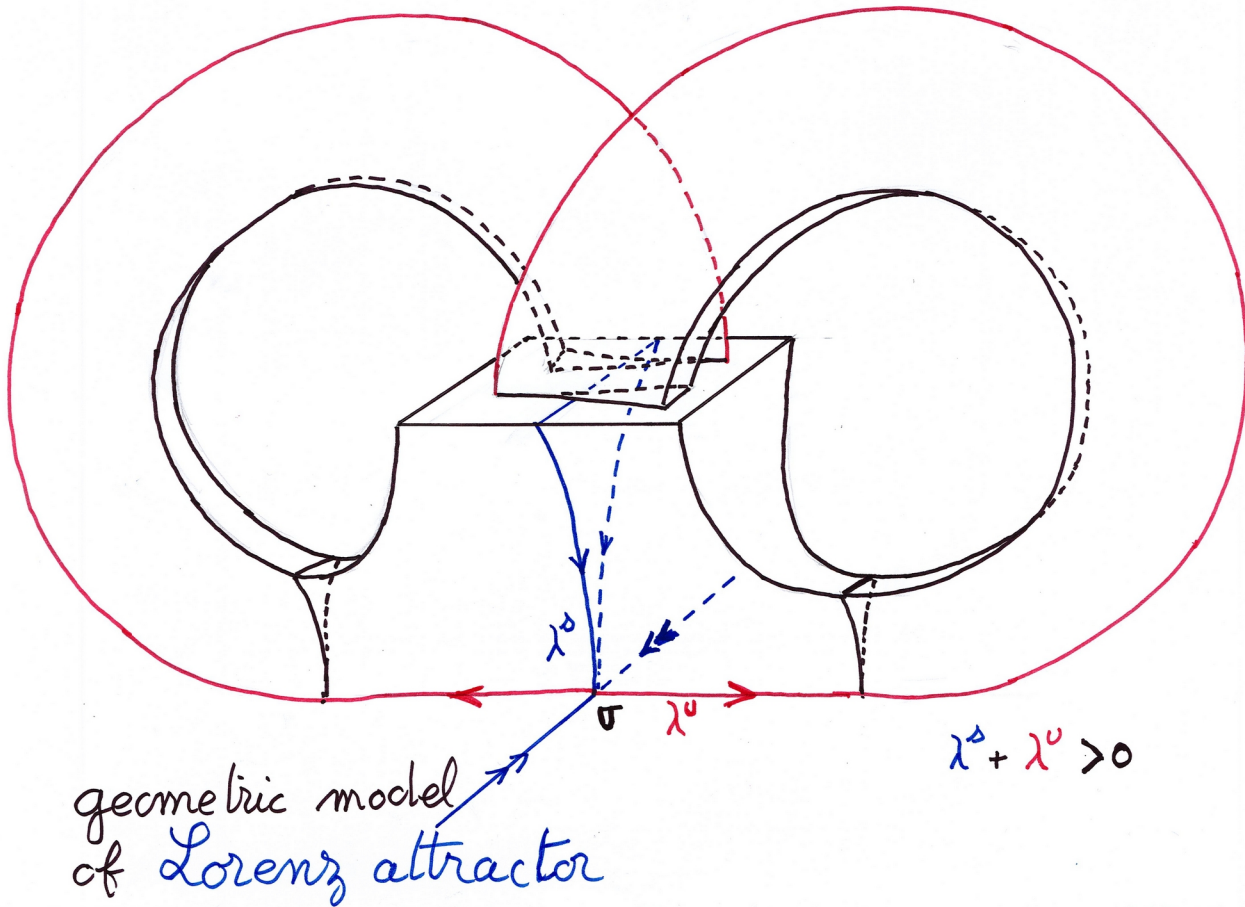


## Singular hyperbolicity



The Lorenz attractor, in its geometric model of Guckenheimer&Williams and Afraimovich&Bikov&Shil'nikov is the first example of  $C^1$  open subset of the space of vector fields, where singular points are accumulated by recurrent orbits, and by periodic saddle-type orbits .

As always, when there is a robust property in  $C^1$  topology, it is because there is a structure on the tangent bundle, a form of hyperbolicity. The difficulty here is to make compatible the hyperbolicity of the regular orbits (with 3 bundles of dimension 1, the stable, the unstable and the direction of the flow) and the hyperbolicity of the singular point, which has only two spaces: the stable and the unstable.

In dimension 3, this difficulty was resolved by the notion of singular hyperbolicity, by Morales Pacifico and Pujals. But the straightforward generalizations of this notion in larger dimensions arbitrarily imposed that the singularities in the same set (transitive or recurrent by chains) all had the same index. That was clearly not satisfactory: With Li Ming and Yi Shi I built a robustly recurring chain attractor containing singularities of different indices [BLY2013] below.

With my student Adriana da Luz, I resolved this difficulty in [BdL2021], using the notion of multisingular hyperbolicity.

## My publications in singular hyperbolicity

1. [BdL2021] Bonatti, Christian; Da Luz, Adriana *Star flows and multisingular hyperbolicity*. **J. Eur. Math. Soc. (JEMS)** 23, No. 8, 2649-2705 (2021).
2. Bonatti, Christian; Pinsky, Tali *Lorenz attractors and the modular surface*. **Nonlinearity** 34, No. 6, 4315-4331 (2021).
3. Barros, Diego; Bonatti, Christian; Pacifico, Maria Jose *Upper, down, two-sided Lorenz attractor, collisions, merging and switching*. ArXiv:2101.07391 **Ergodic Theory and Dynamical Systems**. Published online 2024:1-45. doi:10.1017/etds.2024.8
4. Bonatti, Christian; da Luz, Adriana *Weak hyperbolic structures and robust properties of diffeomorphisms and flows*. Mehrmann, Volker (ed.) et al., *European congress of mathematics. Proceedings of the 7th ECM (7ECM) congress*, Berlin, Germany, July 18–22, 2016. Zürich: **European Mathematical Society (EMS)** (ISBN 978-3-03719-176-7/hbk; 978-3-03719-676-2/ebook). 389-405 (2018).
5. Bonatti, Christian; Gan, Shaobo; Yang, Dawei *Dominated chain recurrent class with singularities*. (English) **Ann. Sc. Norm. Super. Pisa**, Cl. Sci. (5) 14, No. 1, 83-99 (2015).
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7. Bonatti, Christian; Li, Ming; Yang, Dawei *On the existence of attractors*. *Trans. Am. Math. Soc.* 365, No. 3, 1369-1391 (2013).
8. Bonatti, C. *Towards a global view of dynamical systems, for the  $C^1$ -topology*. **Ergodic Theory Dyn. Syst.** 31, No. 4, 959-993 (2011).
9. Bonatti, C.; Grines, V.; Langevin, R. *Dynamical systems in dimension 2 and 3: conjugacy invariants and classification*. **Comput. Appl. Math.** 20, No. 1-2, 11-50 (2001).
10. Bonatti, Ch.; Pumariño, A.; Viana, M. *Lorenz attractors with arbitrary expanding dimension*. Fiedler, B. (ed.) et al., *International conference on differential equations. Proceedings of the conference, Equadiff '99*, Berlin, Germany, August 1-7, 1999. Vol. 1. Singapore: World Scientific. 39-44 (2000).
11. Bonatti, Christian; Pumariño, António; Viana, Marcelo *Lorenz attractors with arbitrary expanding dimension*. **C. R. Acad. Sci.**, Paris, Sér. I, Math. 325, No. 8, 883-888 (1997)