Partially hyperbolic diffeomorphisms

What is it about? We have seen that a dynamic is hyperbolic if at any point the tangent space is the sum of the uniformly contracted directions (stable space) and those uniformly dilated (unstable space). A system is said to be "partially hyperbolic" if the space at any point tangent space is the sum of the stable space (uniformly contracted vectors) of the unstable space (uniformly dilated vectors) and a central space where the Vectors can be contracted or expanded but are uniformly less contracted than those in stable space and uniformly less expanded than those in unstable space. ... There are many partially hyperbolic structures depending on the respective dimensions of the central, stable, and unstable spaces.

Typical examples are
- time 1 of an Anosov flow,
-the fibered product of an Anosov diffeomorphism by rotations
- a linear Anosov diffeomorphism having an eigenvalue closer to 1 than the other two.

The study of these systems became important in particular in the 1990s/2000s with a series of results in two different directions but of the same flavor:
- in ergodic theory, they provided the first examples of "stably ergodic" dynamics: these are volume-preserving diffeomorphisms such that the volume shape remains ergodic for any volume-preserving disturbance.
- in topological dynamics, they provided the first examples of "robustly transitive" systems: these are diffeomorphisms in which all disturbances remain transitive (have dense orbits in the manifold).

Conversely we have shown (Diaz Pujals Ures in dimension 3 then [BDP2003] in all dimensions) that
Theorem *Any robustly transitive system admits a partially hyperbolic structure*.

Partially hyperbolic diffeomorphisms have been studied
   • for global properties in all dimensions: robust transitivity [BD1996] (using blenders), stable ergodicity;
   • for their global properties, on varieties of dimension 3, assuming 3 bundles of dimension 1. The essential question is whether they are comparable in one way or another to one of the model examples. The first notion of comparable was called "central leaf conjugation" and was refuted by a series of examples (by Hertz Hertz Ures then by Potrie Gogolev and me &co see below [BGP2016][BPP2016][BGHP2020]). New notion have been recently proposed (collapsed Anosov flows).
   • for semi-global ergodic properties: existence of attractive measure (SRB measures for Sinais Ruelle Bowen) see [ABV2000] and [BV2000]
   • for local properties essentially ergodic properties: characterization of non-hyperbolicity by the existence of non-hyperbolic measurements, approximation of measurements by periodic orbits, etc. with the use of Blenders in an essential way.

My publications in these topics:

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