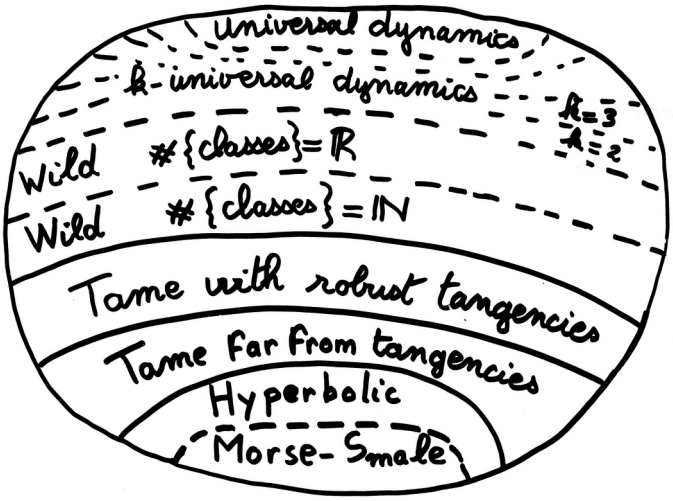
Generic dynamics

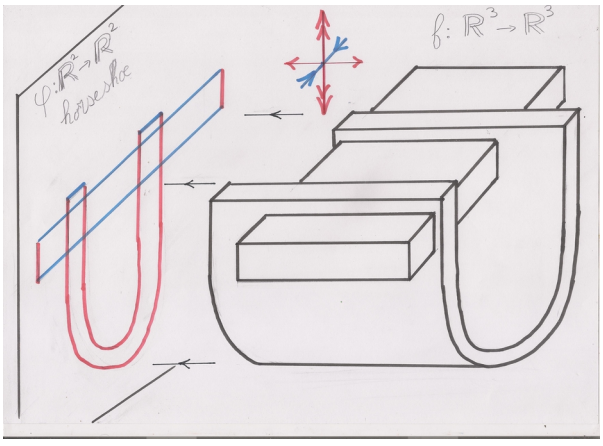
We cannot describe all the possible behavior of the dynamics of diffeomorphisms or flows: there are too many. But perhaps the most complex behaviors are carried out by very degenerate systems. A bit like what happens with Morse functions: the singular points of any function can be very complex but, considering a dense&open subset of the set of functions, they only have a finite set of singularity which are moreover non-degenerate.

This is the approach that Smale had attempted with hyperbolic systems, but which quickly proved not to describe a dense family. A large part of my work in dynamic systems (57 references below) consists of describing the dynamics of diffeomorphisms in an open and dense, or generic, part (intersection of a countable family of dense open) of the set of all the systems. For this to make sense, it is necessary to specify the topology on the space of diffeomorphisms, and my work mainly concerns the C^1 topology.

My work (with numerous collaborators, Diaz and Crovisier being the main ones) allowed me to develop a conjectural mapping of a dense open space of all diffeomorphisms, for the C^1 topology. This is an ambitious program which brings together the results already obtained using around twenty conjectures, none of which has yet been refuted. (Reference 25 below, 2011).



Among my works that have marked this subject, there is the definition, construction and consequences of “Blenders” with Lorenzo Diaz. These blenders (reference 56, 1996) remained misunderstood and neglected by the community for around ten years, before the communities took hold of them, used them, adapted them to other contexts (conservative, symplectic, holomorphic) generalize them like Berger's notion of parablenders. This is one of the notions that I would have introduced (not alone of course) and which will be integrated into the scientific tools well beyond the initial subject, and certainly beyond mathematics.



There is also the generalization, with Crovisier, of the “Hayashi connecting lemma” (2004, reference 42) and which made it possible to use Conley's theory on chain recurrence classes for C1-generic diffeomorphisms.

There is also numerous work on the perturbations of the differential along periodic orbits, in line with the work of Mañé and Liao, which makes it possible to show that weak (but uniform) notions of hyperbolicity are necessary for existence of robust properties (see in particular reference 48 with Diaz and Pujals, in 2003).

My most recent work focused on the characterization of non-hyperbolic dynamics by the existence of measures with zero Lyapunov exponents… using Blenders. My latest work, with Diaz and Gelfert (reference 1), is in this line.

Finally, let us mention the book ***Dynamics beyond uniform hyperbolicity. A global geometric and probabilistic perspective*** with Diaz and Viana, who gave its name *Dynamics beyond uniform hyperbolicity* to a long series of Conferences, which was the main event in the field.

List of my publications for the study of C^1-robust or generic properties.

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2. Bonatti, Christian; Shinohara, Katsutoshi *Aperiodic chain recurrence classes of C^1-generic diffeomorphisms.* Preprint, arXiv:2209.13249 (2022).
3. Bonatti, Christian; Shinohara, Katsutoshi *A mechanism for ejecting a horseshoe from a partially hyperbolic chain recurrence class.* ArXiv:2209.13245 **Ergodic Theory and Dynamical Systems.** Published online 2023:1-63. doi:10.1017/etds.2023.76
4. Bonatti, Christian; Díaz, Lorenzo J.; Kwietniak, Dominik *Robust existence of nonhyperbolic ergodic measures with positive entropy and full support.* **Ann. Sc. Norm. Super. Pisa**, Cl. Sci. (5) 22, No. 4, 1643-1672 (2021).
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